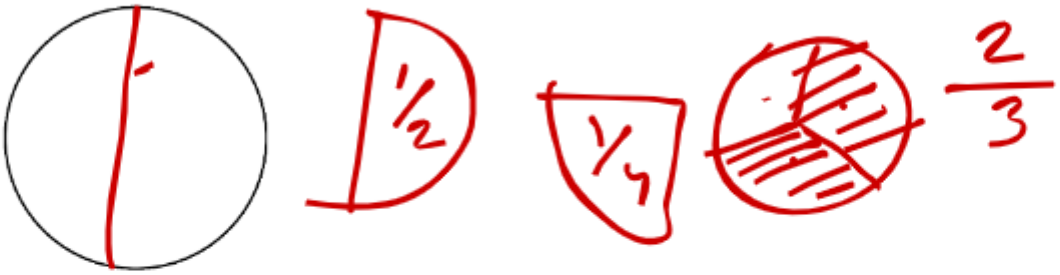
FRACTIONS

Any part of a whole number can be expressed in terms of fractions. These can be categorised into 3 different forms: Common or Vulgar fractions, Decimal fractions, and Percentile fractions.



Vulgar or Common Fractions have an integer on the numerator and an integer on the denominator.

$$\frac{5}{6} \text{ or } \frac{7}{15}$$

These can further be sub-divided into Proper fractions, Improper fractions, and Mixed fractions.

Proper fractions are those that have a numerator smaller than the denominator.

$$\frac{3}{11}$$

Improper fractions have a numerator greater than the denominator.

$$\frac{7}{5} = 1\frac{2}{5}$$

Mixed fractions are whole numbers plus a fractional part associated with it.

$$2\frac{3}{8}$$

Converting Mixed into improper fractions and vice versa:

$$2\frac{3}{8} = \frac{19}{8}$$

$$11\frac{2}{5} = \frac{57}{5}$$

$$\frac{9}{7} = 1\frac{2}{7}$$

$$\frac{35}{8} = 4\frac{3}{8}$$

A **Decimal fraction** is a fraction with multiples of 10 in the denominators, expressed with a decimal. The numbers after the decimal represent the fractional part.

$$\frac{25.72}{11} = \frac{2572}{100}$$

$$\frac{3}{10} = 0.3$$

A **Percentile fraction** is a fraction with 100 in the denominator, expressed with a % sign.

$\frac{15}{100} \longrightarrow 15\%$

Reciprocal of a common fraction is the numerator and denominator interchanged.

Reciprocal of $\frac{5}{8}$ is $\frac{8}{5}$ $\quad \frac{1}{2/7} = \frac{7}{2}$

Addition of fractions

Addition of two fractions

1) When the denominators are the same, add the numerators

$\frac{5}{11} + \frac{3}{11} = \frac{8}{11}$

$\frac{7}{17} + \frac{2}{17} + \frac{3}{17} = \frac{12}{17}$

2) When the denominators can be made same by simple multiplication

$\frac{5}{18} + \frac{1}{6}$

$\frac{5}{18} + \frac{3}{18} = \frac{8}{18} = \frac{4}{9}$

Two fractions

3) When the numerator is 1 in both fractions

$\frac{1}{4} + \frac{1}{13}$

Add the denominators for the numerator of the result, $13 + 4 = 17$

Multiply the denominators for the denominator of the result, $13 \times 4 = 52$

$\frac{17}{52}$

$4 \times 13 = 52$

$\frac{1}{11} + \frac{1}{13} = \frac{24}{143}$

$3\frac{1}{5} + 5\frac{1}{8} = 8\frac{13}{40}$

4) When none of the above conditions fit, then cross multiply and add the results for the numerator and multiply both denominators for the resultant denominator.

$$\frac{5}{11} \times \frac{3}{7}$$

$$= \frac{5(7)+3(11)}{11 \times 7} = \frac{35+33}{77} = \frac{68}{77}$$

$$3\frac{4}{7} + 4\frac{2}{5}$$

$$7\frac{20+14}{35} = 7\frac{34}{35}$$

Subtraction of fractions:

Similar to the above procedure

$$3\frac{5}{7} - 1\frac{1}{3}$$

$$2\frac{15-7}{21} = 2\frac{8}{21}$$

Multiplication of fractions:

In case of multiplication, any number on the numerators can be divided by (cancel proportionately) any number on the denominators.

$$\frac{5}{8} \times \frac{4}{21} \times \frac{42}{55}$$

$$= \frac{5}{8} \times \frac{4}{21} \times \frac{42}{55}$$

$$= 1 \times 1 \times \frac{1}{11} = \frac{1}{11}$$

$$= \frac{1}{1} \times \frac{1}{1} \times \frac{1}{11} = \frac{1}{11}$$

$$3\frac{5}{7} \times 1\frac{1}{2} \times 2\frac{2}{13}$$

$$\frac{26}{7} \times \frac{3}{2} \times \frac{28}{13} = \frac{12}{1} = 12$$

Division of fractions:

For division of fractions, flip the dividing fraction after switching the division sign with that of multiplication.

$\frac{4}{9} \div \frac{8}{11}$

Becomes $\frac{4}{9} \times \frac{11}{8} = \frac{11}{18}$

$4\frac{2}{17} \div 1\frac{3}{17}$

$\frac{70}{17} \div \frac{20}{17} = \frac{70}{17} \times \frac{17}{20} = \frac{7}{2} = 3\frac{1}{2}$

Decimal fractions:

Every rational number is either a terminating decimal or a recurring decimal. A fraction in lowest terms with a prime denominator other than 2 or 5 always produces a repeating (recurring) decimal.

Note that 22/7 is a rational number but π is not, as 22/7 is an approximation of the irrational number π.

1) ***Terminating decimals:***

As the name suggests, in this case, the decimal value terminates after a certain number of digits. 15.58, 3.232, etc. are terminating decimals.

A fraction results in a terminating decimal only if the numerator is divided by any multiple of 2 and/or 5.

$$\frac{3}{16} = \frac{3}{2^4} = 0.1875$$

$$\frac{13}{50} = \frac{13}{2 \times 5^2} = 0.26$$

2) ***Recurring decimals:***

Also expressed by a bar on the recurring numbers, these decimals do not terminate but are continuous infinitely.

$$\frac{1}{3} = 0.\overline{3} = 0.333333 \ldots$$

$$\frac{3}{7} = 0.\overline{428571} = 0.42857142857 \ldots\ldots$$

$$\frac{1}{81} = 0.\overline{012345679} = 0.0123456790123 \ldots$$

$$\frac{7}{12} = 0.58\overline{3} = 0.583333\ldots$$

CYCLIC NUMBERS

If the repetend length of 1/p for prime p is equal to p − 1 then the repetend, expressed as an integer, is called a cyclic number. For example

$\frac{1}{7} = 0.\overline{142857}$ (The recurring numbers are 6, one less than 7. Therefore this is an example of a cyclic number)

$\frac{1}{17} = 0.\overline{0588235294117647}$ (16 recurring digits, therefore a cyclic number)

Converting recurring decimals to fractions:

1) When all digits are recurring:
   Divide the digits by an equal number of 9s.

$$0.\overline{56} = 0.565656\ldots. = \frac{56}{99}$$

$$0.\overline{347} = 0.347347\ldots = \frac{347}{999}$$

$$0.\overline{5324} = \frac{5324}{9999}$$

2) When some digits are not recurring:

$$0.3\overline{4} = 0.3444444\ldots. = \frac{34-3}{90} = \frac{31}{90}$$

$$0.5\overline{789} = 0.5789789\ldots = \frac{5789-5}{9990} = \frac{5784}{9990} = \frac{964}{1665}$$

$$0.23\overline{456} = 0.23456456\ldots. = \frac{23456-23}{99900}$$

$$\frac{23433}{99900} \neq \frac{7811}{33300}$$

Equivalence table for different fractions:

Common Fractions	Decimal Fractions	Percentile Fractions
$\frac{1}{2}$	0.5	50%
$\frac{1}{3}$	$0.\overline{3}$	33.33%
$\frac{1}{4}$	0.25	25%
$\frac{1}{5}$	0.2	20%
$\frac{1}{6}$	$0.1\overline{6}$	16.67%
$\frac{1}{8}$	0.125	12.5%
$\frac{3}{4}$	0.75	75%
$\frac{2}{5}$	0.4	40%
$\frac{3}{5}$	0.6	60%

$75\% \times 16 = \frac{3}{\cancel{4}} \times \overset{4}{\cancel{16}} = 12$

$0.6 \times 50 = \frac{3}{5} \times \overset{10}{\cancel{50}} = 30$