

## Arithmetic Sequence

A set of numbers that are equally spaced, with a fixed difference between two consecutive members, is called an Arithmetic Sequence. Such as

$10, 13, 16, 19, 22, 25, \dots$   $\rightarrow d = +3$   
 $45, 41, 37, 33, 29, \dots$   $\rightarrow d = -4$

1. The Arithmetic Mean,  $M_n$ , of  $n$  terms of an Arithmetic Sequence is calculated by

$$M_6 = \frac{10 + 25}{2} = 17.5 \quad M_n = \frac{a_1 + a_n}{2} \quad M_5 = \frac{45 + 29}{2} = 37$$

2. Any term of an Arithmetic Sequence can be calculated by using the formula

$$a_n = a + (n - 1)d$$

Where  $a_n$  is the  $n^{\text{th}}$  term,  $a$  is the first term,  $n$  is the number of terms, and  $d$  is the common difference.

For example: In the sequence

$45, 41, 37, 33, 29, \dots, 25, 21, 17, 13$

$$a = 45$$

$$d = 41 - 45 = -4$$

If one wants to find out the value of the 9<sup>th</sup> term,  $a_9$ , then  $n = 9$ .

Using the formula

$$a_9 = 45 + (9 - 1)(-4)$$

$$a_9 = 45 - 32$$

$$a_9 = 13$$

3. The sum,  $S_n$ , of any  $n$  terms of an Arithmetic sequence is given by

$$S_n = \frac{n}{2} \{2a + (n - 1)d\}$$

$$\text{Or } S_n = nM_n$$

$$S_9 = \frac{9}{2} \{2(45) + (9 - 1)(-4)\} = \frac{9}{2} \{90 - 32\} = 261$$

$$S_9 = 9M_9 = 9\left(\frac{45 + 13}{2}\right) = \frac{9(58)}{2} = 261$$

4. The sum of first  $n$  positive integers is

$$S = \frac{n(n + 1)}{2}$$

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + \dots + 100$$

$$S = \frac{100(101)}{2} = 5050$$

5. The sum of first  $n$  positive odd numbers is equal to  $n^2$ .

$$1 + 3 + 5 + 7 + 9 + 11 + 13 = 7^2 = \underline{\underline{49}}$$

6. The last term,  $a_n$ , of first  $n$  positive odd integers is equal to  $2n - 1$ .

$$a_7 = 2(7) - 1 = 14 - 1 = \underline{\underline{13}}$$

7. The sum of first  $n$  even numbers is equal to  $n(n + 1)$ .

$$2 + 4 + 6 + 8 + 10 + 12 + 14 = \cancel{14(15)} = \cancel{210}$$
$$7(8) = 56 = \underline{\underline{210}}$$

8. The last term,  $a_n$ , of first  $n$  positive even integers is equal to  $2n$ .

$$a_7 = 2(7) = 14$$

9. In an arithmetic sequence with an odd number of members, the average of the members is the middle term.

$$82, 87, \textcircled{92}, 97, 102$$

↑