

Arithmetic Sequence

A set of numbers that are equally spaced, with a fixed difference between two consecutive members, is called an Arithmetic Sequence. Such as

$$10, 13, 16, 19, 22, 25, \dots \quad d = +3$$

$$45, 41, 37, 33, 29, \dots \quad d = -4$$

1. The Arithmetic Mean, M_n , of n terms of an Arithmetic Sequence is calculated by

$$M_6 = \frac{10 + 25}{2} = 17.5$$

$$M_n = \frac{a_1 + a_n}{2}$$

$$M_5 = \frac{45 + 29}{2} = 37$$

2. Any term of an Arithmetic Sequence can be calculated by using the formula

$$a_n = a + (n - 1)d$$

Where a_n is the n^{th} term, a is the first term, n is the number of terms, and d is the common difference.

For example: In the sequence

$$45, 41, 37, 33, 29, \dots, 25, 21, 17, 13 \quad \leftarrow$$

$$a = 45$$

$$d = 41 - 45 = -4$$

If one wants to find out the value of the 9^{th} term, a_9 , then $n = 9$.

Using the formula

$$a_9 = 45 + (9 - 1)(-4)$$

$$a_9 = 45 - 32$$

$$a_9 = 13$$

3. The sum, S_n , of any n terms of an Arithmetic sequence is given by

$$S_9 = \frac{9}{2} \{2(45) + (9-1)(-4)\} = \frac{9}{2} \{90 - 32\} = 261$$

$$S_9 = 9M_9 = 9 \left(\frac{45 + 13}{2} \right) = \frac{9(58)}{2} = 261 \quad \leftarrow$$

4. The sum of first n positive integers is

$$S = \frac{n(n + 1)}{2}$$

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + \dots + 100 \quad S = \frac{100(101)}{2}$$

$$= \underline{\underline{5050}}$$

5. The sum of first n positive odd numbers is equal to n^2 .

$$1+3+5+7+9+11+\underline{13}=7^2=\underline{\underline{49}}$$

6. The last term, a_n , of first n positive odd integers is equal to $2n - 1$.

$$a_7 = 2(7) - 1 = 14 - 1 = \underline{\underline{13}}$$

7. The sum of first n even numbers is equal to $n(n + 1)$.

$$2+4+6+8+10+\cancel{12}+\cancel{14} = \cancel{11}(\cancel{15}) = \cancel{22}4$$
$$7(8) = 56 = \cancel{2}4$$

8. The last term, a_n , of first n positive even integers is equal to $2n$.

$$a_7 = 2(7) = 14$$

9. In an arithmetic sequence with an odd number of members, the average of the members is the middle term.

$$82, 87, \textcircled{92}, 97, 102$$