

CONSECUTIVE INTEGERS

Integers written in the normal order, one after the other, are consecutive integers.

$$\dots \underline{-3, -2, -1, 0, 1, 2, 3, 4, 5, \dots}$$

Some interesting facts about consecutive integers:

1. The sum of a list of consecutive integers can be calculated by multiplying the average of the numbers by the number of integers. For example

$$\text{The average of terms is } \frac{-1+3}{2} = 1 \text{ and there are } 5 \text{ terms. So the sum must be equal to } 1 \times 5 =$$

$$\text{Avg} = \frac{-1+3}{2} = \frac{2}{2} = 1 \quad S = 5 \times 1 = 5$$

2. The product of x consecutive integers is perfectly divisible by $x!$.

Example:

$$\text{The product is } 2 \times 3 \times 4 \times 5 \times 6 \times 7 = 5040$$

Whereas $6! = 720$
And $5040/720 = 7$

$$\frac{5040}{720} = 7$$
$$x = 6 \quad 6! \\ 6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 \\ = 720$$

3. Any pair of consecutive integers has one even and one odd integer.

$$\cancel{56, 57, 58, 59, 60}, 61, 62$$

4. The product of any two consecutive integers is always even.

$$100 \times 101 = E \\ 57 \times 58 = E$$

5. Any 3 consecutive integers always have one integer divisible by 3.

6. The product of any 3 consecutive integers is always divisible by 2, 3, and 6.

$$\frac{58 \times 59 \times 60}{2^1 \quad 3^1}$$

7. The product of any 5 consecutive integers is always divisible by 2, 3, 4, 5, and 6.

$$101 \cancel{102} \times 103 \times 104 \times 105 \quad 12, 15$$