Factors and Multiples

A factor or a divisor d, of a non-zero integer x, divides x into f integers, without leaving any remainder. Mathematically,

x = fd (Where all x, f, and d are all integers.)

All integers are factors of themselves.

1 and – 1 are factors of all integers.

0 can be divided by all integers, except by 0 itself.

0 divided by 0 is undefined.

If x is a factor of m and also a factor of n, then x is a factor of $(m + n)$. x is also a factor of $(ma + nb)$, where a and b are integers.

If x is a factor of y and y is a factor of z, then x must be a factor of z.

If two integers, x and y, are factors of each other, then either x = y, or x = – y.

If a prime number x is a factor of a product of two numbers a and b, then x is a factor of a or a factor of b.

A **Proper Divisor** is an integer that divides any non-zero integer and is not equal to the integer itself.

For example, in case of 12, 1, 2, 3, 4, and 6 are proper divisors.

A Prime number has exactly two factors.

The **number of factors of an integer** can be found by using prime factorization. If a number, after prime factorization, is $p = x^a y^b z^c$, then the number of its factors is given by

$$(a+1)(b+1)(c+1).$$

For example: $720 = 2^4 3^2 5^1$, then 720 has $(4+1)(2+1)(1+1) = 30$ factors. Those 30 factors are {1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 16, 18, 20, 24, 30, 36, 40, 45, 48, 60, 72, 80, 90, 120, 144, 180, 240, 360, 720}

$12 = 2 \times 2 \times 3 = 2^2 3^1$ $\quad (2+1)(1+1) = 3 \times 2 = 6$

$12 \rightarrow \{1, 2, 3, 4, 6, 12\}$

The **sum of all factors of an integer**, p, is given by the formula

$$S = \frac{(x^{a+1}-1)(y^{b+1}-1)(z^{c+1}-1)}{(x-1)(y-1)(z-1)}, \text{ where } p = x^a y^b z^c$$

$12 = 2^2 3^1$

For example: $180 = 2^2 3^2 5^1$, then the sum of all factors of 180 is equal to $S = \frac{(2^{2+1}-1)(3^{2+1}-1)(5^{1+1}-1)}{(2-1)(3-1)(5-1)} = \frac{(8-1)(27-1)(25-1)}{(1)(2)(4)} = \frac{4368}{8} = 546$

$x = 2$, $a = 2$, $y = 3$, $b = 1$

$S = \frac{(2^3-1)(3^2-1)}{(2-1)(3-1)} = \frac{7 \times 8}{2} = 28$

A **Perfect Number** is a positive integer that is the sum of all its proper factors. For example 6 is a perfect number as 6 = 1 + 2 + 3, so is 28 as 28 = 1 + 2 + 4 + 7 + 14

EUCLID's FORMULA:

If $2^n - 1$ is a prime number, then $(2^n - 1)2^{n-1}$ is a Perfect Number.

Examples: $2^2 - 1$ *is* 3, *a prime number, so* $(2^2 - 1)2^{2-1} = 6$ is a perfect number and

$2^3 - 1$ *is* 7, *a prime number, so* $(2^3 - 1)2^{3-1} = 28$ is a perfect number.

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